

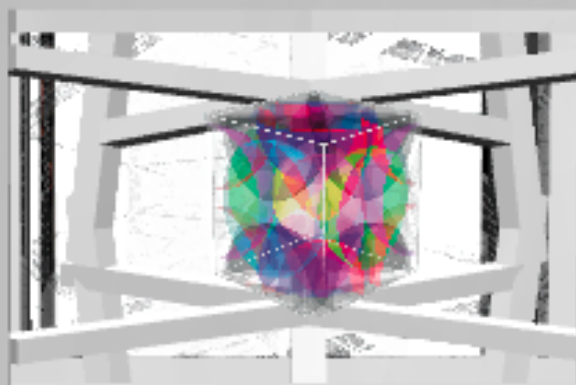
Um Olhar
nas Espaços de
Dimensão 3

Sala



10 de fevereiro de 2014
10 de fevereiro de 2014
10 de fevereiro de 2014

Um Olhar
nos Espaços de
Dimensão 3



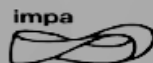
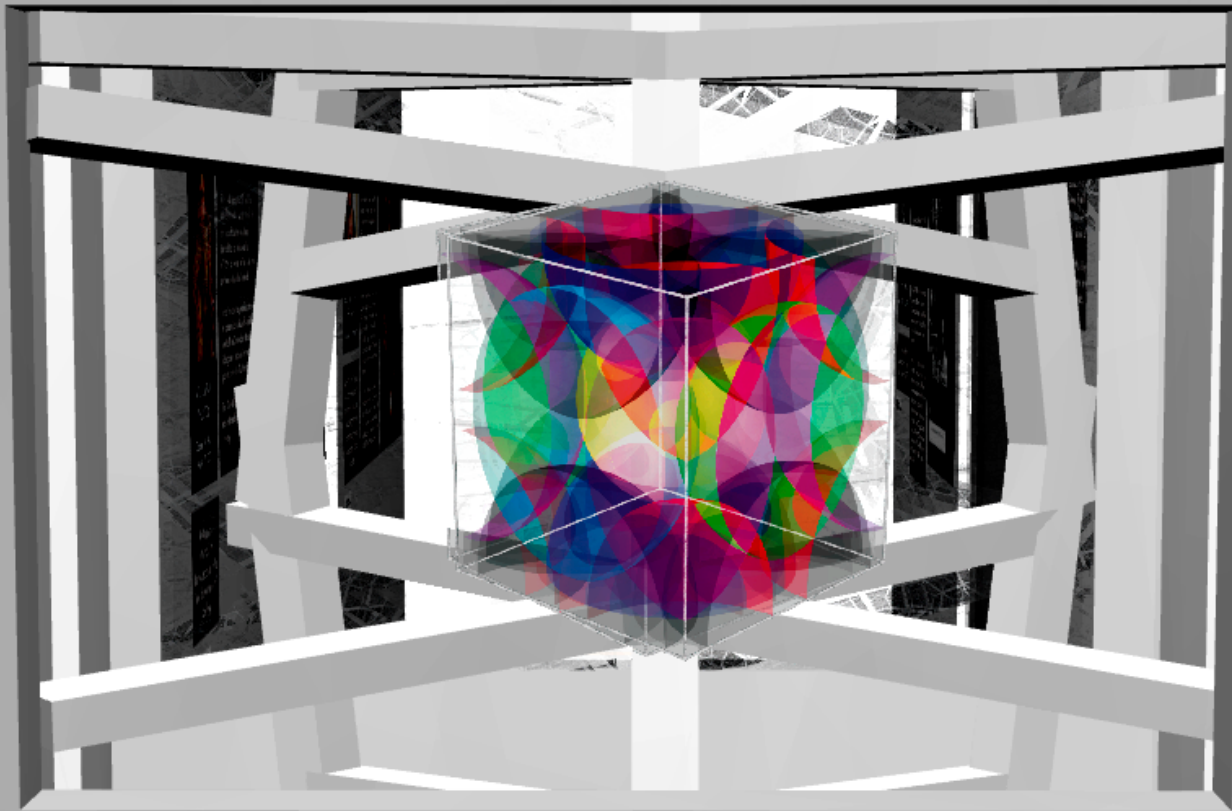
UNIVERSIDADE
DE SÃO PAULO

USP

INSTITUTO
DE MATEMÁTICA

DE SÃO CARLOS

BRASIL



Geometria e Topologia

ESFERA

DOMÍNIO FUNDAMENTAL

ESPAÇOS

MÉTRICA

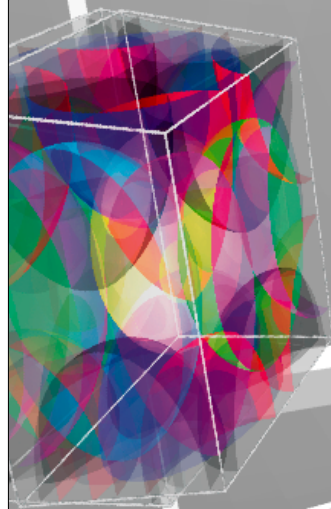
GEODÉSICA

CURVATURA

ESFERA

Visualização Interior de Variedades 2D

Mapa

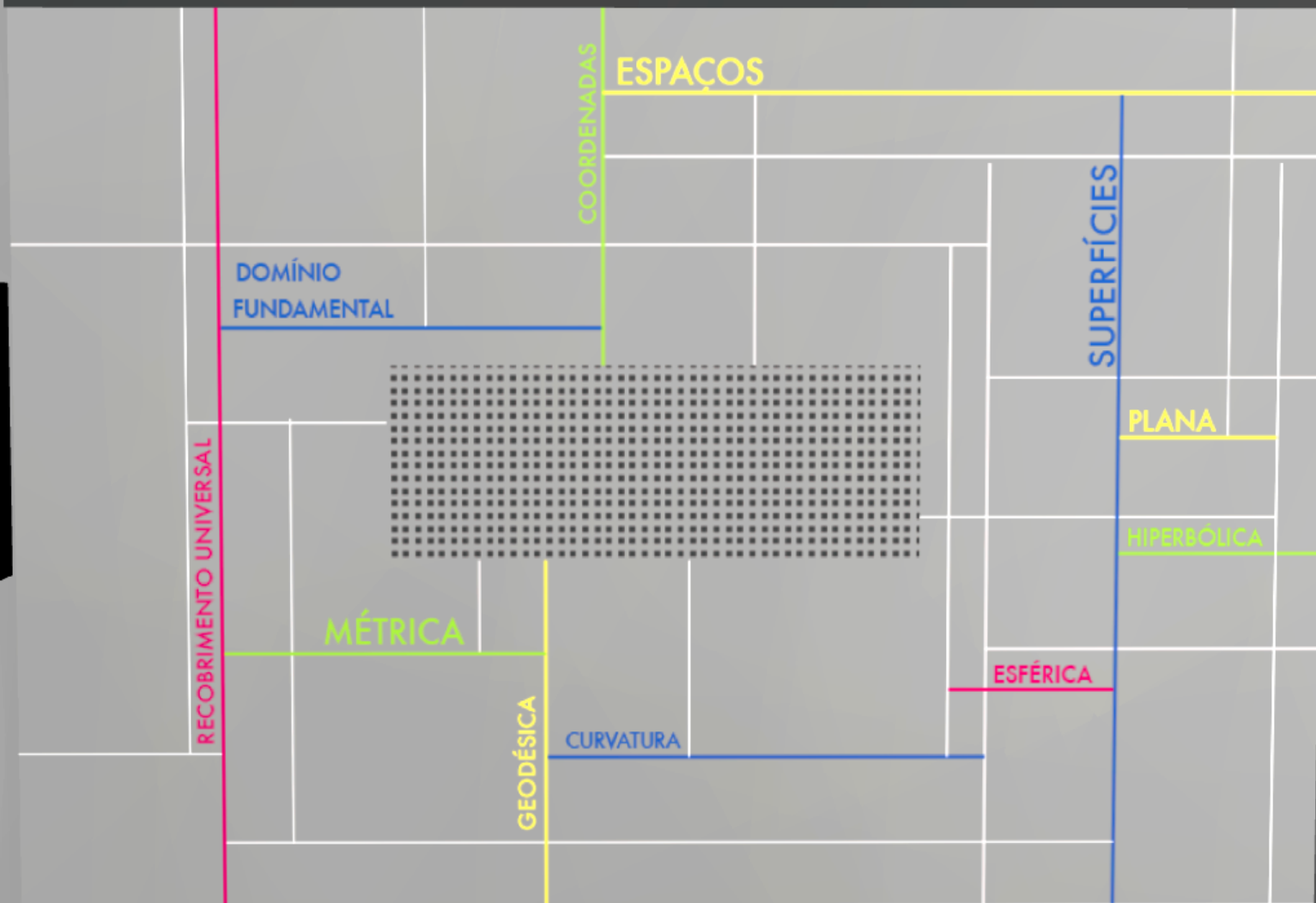


BR&SIL



Centro Brasileiro de Pesquisas Físicas

Geometria e Topologia



VARIEDADES

Variedades e Dimensão

Para um ponto, os pontos adjacentes
em uma linha se seguem.

Uma curva é uma
variedade de dimensão 1

Em uma superfície, os pontos adjacentes
em um plano formam um plano.

Uma superfície é uma
variedade de dimensão 2

Para uma variedade, os pontos adjacentes
em um espaço formam um espaço.

Variedade
de dimensão 3

Classificação de Superfícies

“ Qualquer
sistema de
variáveis
inicialmente
provavelmente
solução de

No resumo
analisar de
casos um
problema
topológico

(M. Maria)

Geometria Simplicial

Um ponto é um
simplex de dimensão 0

Um segmento de reta é um
simplex de dimensão 1

Um triângulo é um
simplex de dimensão 2

Um tetraedro é um
simplex de dimensão 3

DIMENSÃO

DIMENSÃO

Geometria Simplicial



Um ponto é um
simplexo de dimensão 0



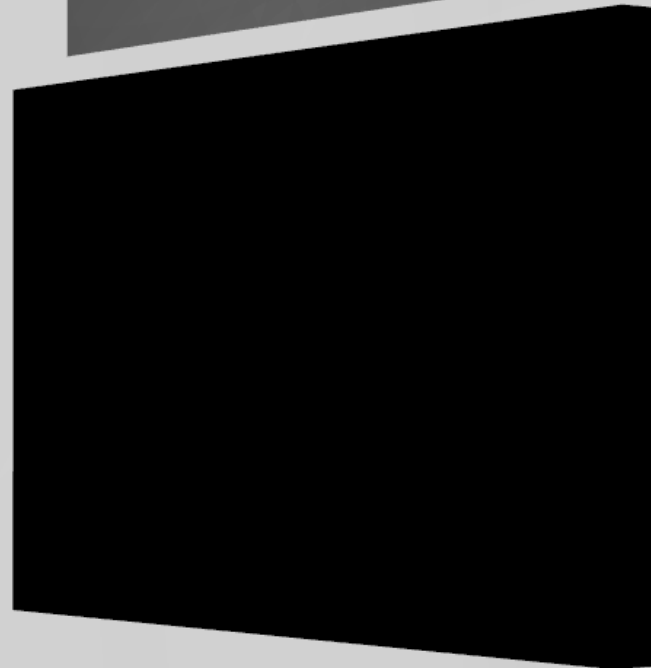
Um segmento de reta é um
simplexo de dimensão 1



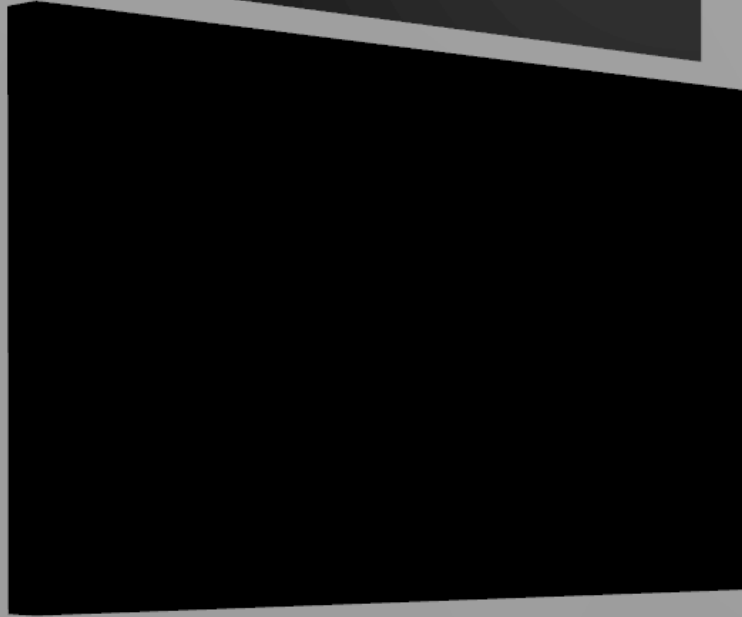
Um triângulo é um
simplexo de dimensão 2



Um tetraedro é um
simplexo de dimensão 3



VARIEDADES



Variedades e Dimensão



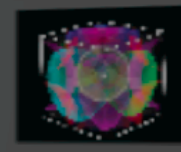
Em uma curva, os pontos vizinhos e os pontos livres são iguais.

Uma curva é uma variedade de dimensão 1



Em uma superfície, os pontos vizinhos e os pontos livres são diferentes.

Uma superfície é uma variedade de dimensão 2



Em uma 3-variedade, os pontos vizinhos e os pontos livres são totalmente diferentes.

Variedade de dimensão 3

DIMENSÃO

Atualização Exterior de Sup

VISÃO INTERIOR/EXTERI

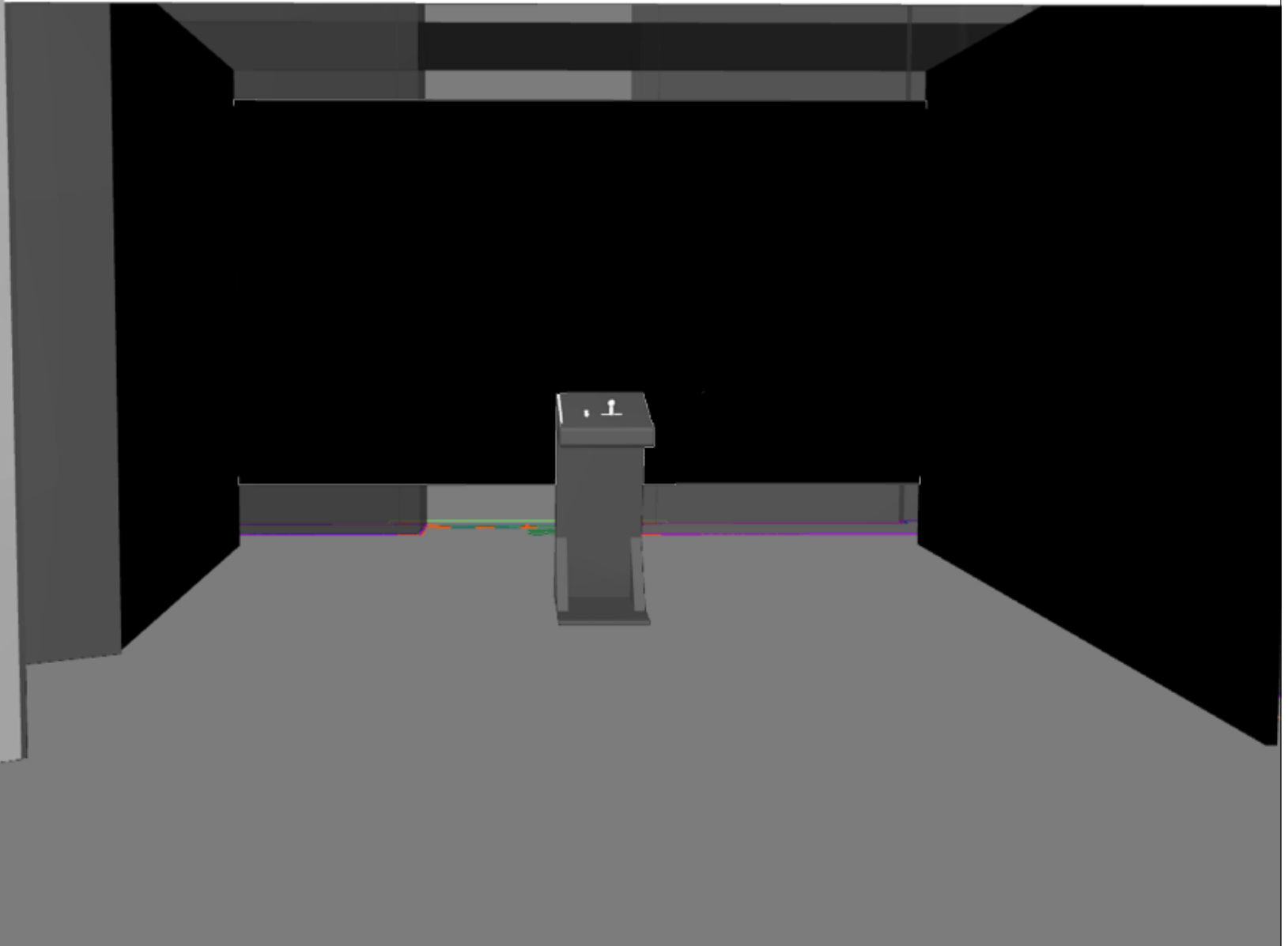


Visualização Exterior de Superfícies

geometria

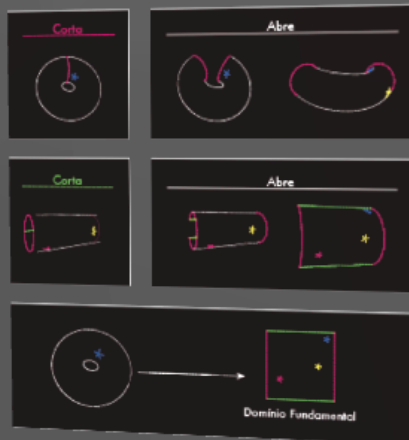


a partícula
superfície

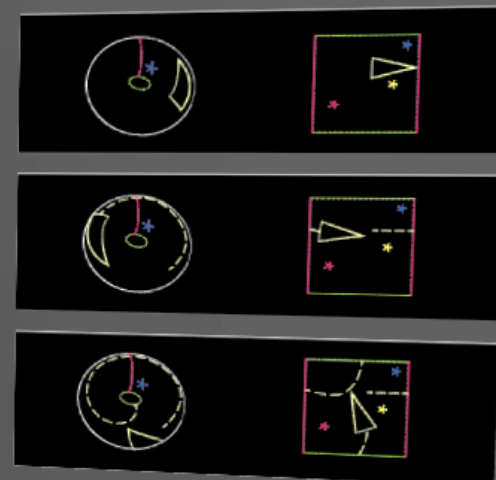


r de Superfícies

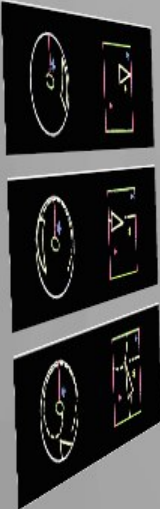
Da variedade ao domínio fundamental



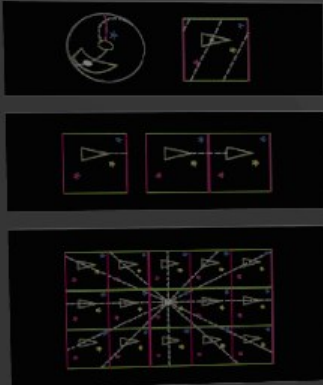
Movimento na variedade



Movimento na variedade



Espaço de recobrimento



VARIETADES

Um raio de luz toma uma trajetória chamada geodésica



A geodésica é o caminho mais curto



Um raio de luz toma uma trajetória chamada geodésica



A geodésica é o caminho mais curto



escolha sua geometria

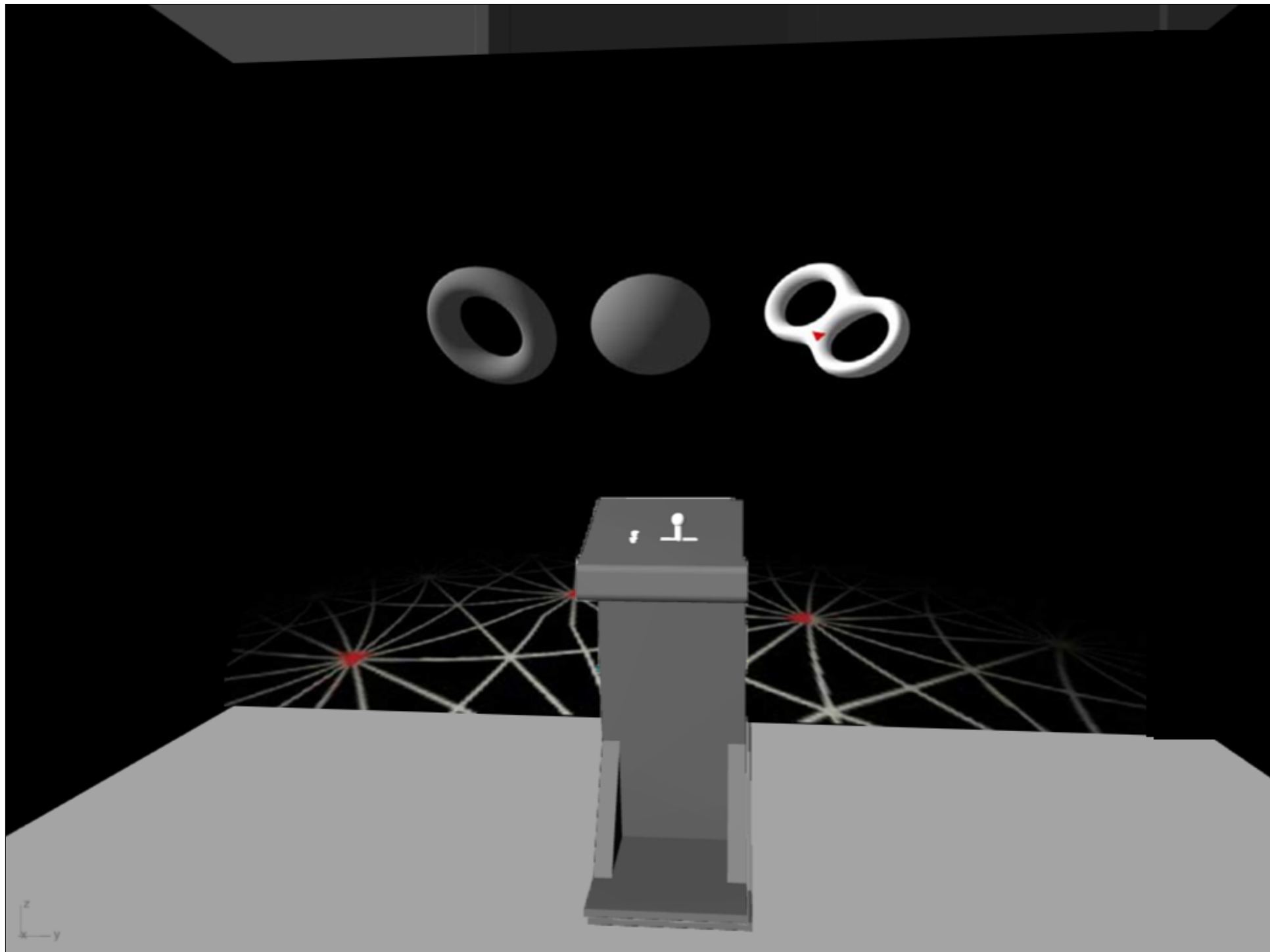


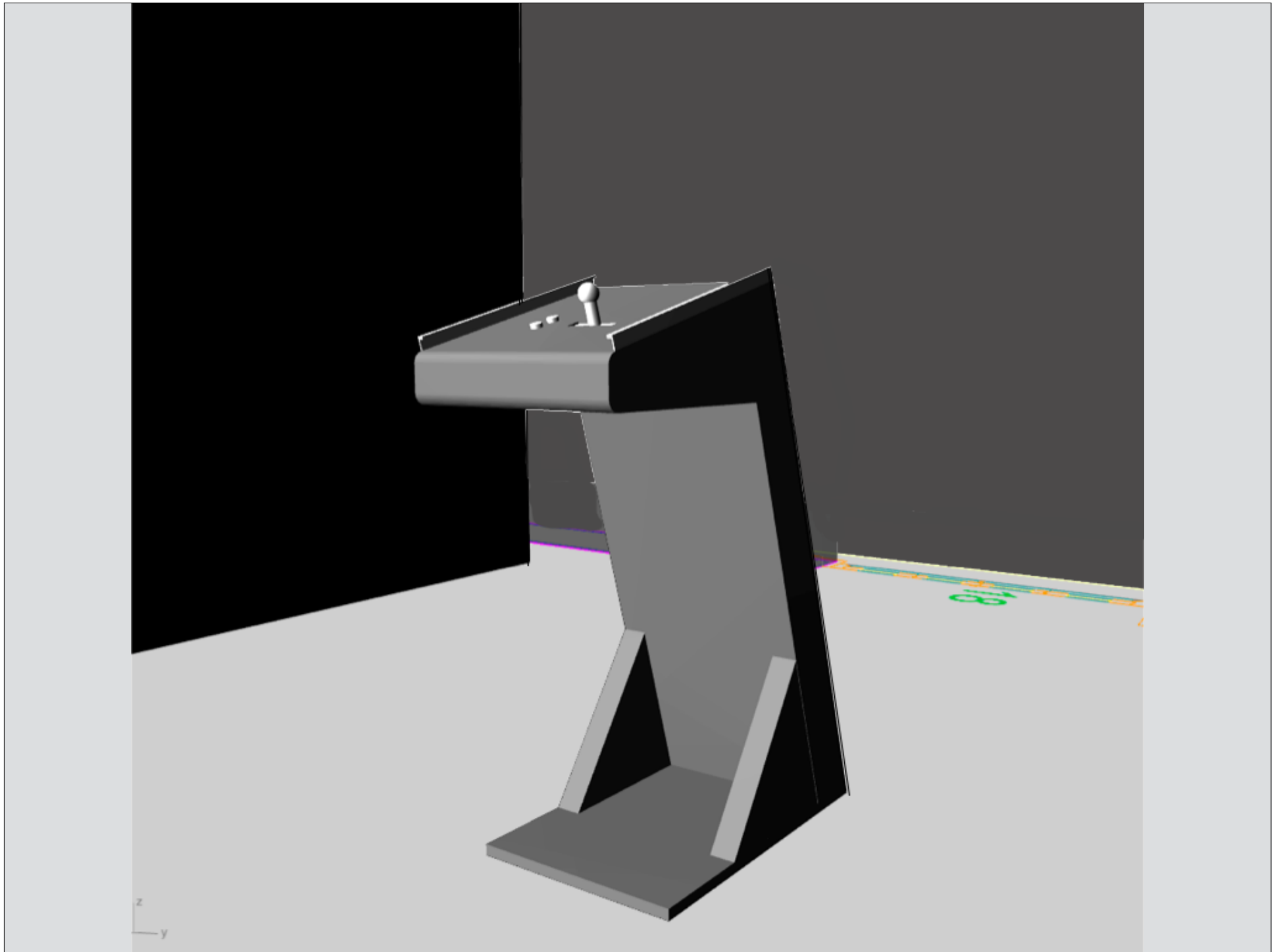
e controle a partícula na superfície



z

y





70s, was made in
a musical instrument.

In mid-1970s,
they played a lot
and theory of
a student.
Felix Mendel for his
workshop.

Algebra complexity
of formulas which
involved a certain
in keeping with
in most general case.

He is professor of
math at Cornell.

erico
de





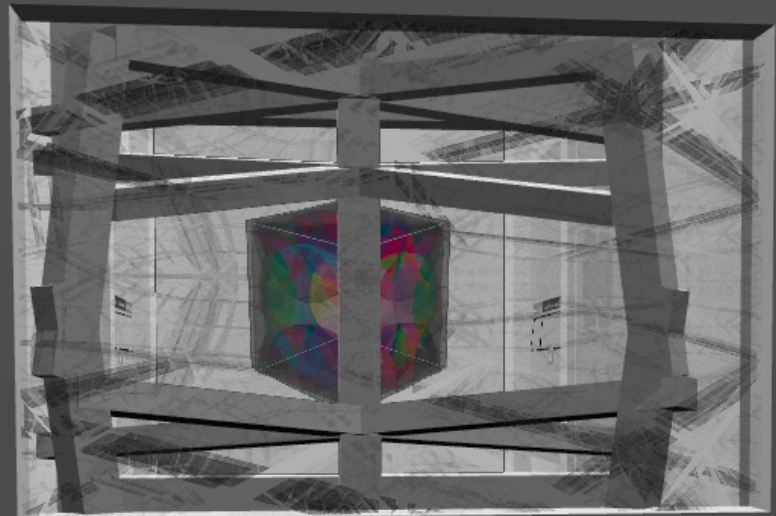
**HAROLD CALVIN
MARSTON MORSE**

(24 de Março de 1892
- 22 de Junho de 1977)

Ele definiu o termo ponto de sela e desenvolveu a teoria dos pontos críticos, incluindo o Teorema do Índice de Morse, que relaciona o número de pontos críticos de uma função com o número de componentes de uma variedade.

Em 1935, Morse foi agraciado com o Prêmio Brouwer de Matemática por seu trabalho na análise funcional.

Matemático Americano
mais conhecido pelo
seu trabalho sobre o
cálculo das variações.



**WILLIAM PAUL
THURSTON**

(October 30, 1946 -
August 21, 2012)

An American
mathematician.
He was a pioneer in the
field of low-dimensional
topology.

His early work, in the early 1970s, was mainly in
Kleinian theory, where he had a dramatic impact.

His later work, starting around the mid-1970s,
revealed that hyperbolic geometry played a far
more important role in the general theory of
3-manifolds than was previously realized.

In 1982, he was awarded the Fields Medal for his
contributions to the study of 3-manifolds.

Thurston formulated the geometrization conjecture.
This gave a conjectural picture of 3-manifolds which
indicated that all 3-manifolds admitted a certain
kind of geometric decomposition involving eight
geometries, now called Thurston model geometries.

From 2001 until his death he was a professor of
mathematics and computer science at Cornell
University.



GRIGORI YAKOVLEVICH PERELMAN
(nascido em 13 de Junho de 1966)

Grigori Yakovlevich Perelman is a Russian mathematician who made landmark contributions to Riemannian geometry and geometric topology before his presumed withdrawal from mathematics.

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In 1994, Perelman proved the real conjecture. In 2001, he proved Thurston's geometrization conjecture. This conjecture is considered the most important and difficult open problem in topology.

In August 2006, Perelman was awarded the Fields Medal [1] for "his contributions to geometry and his revolutionary insights into the analysis and geometric structure of the Ricci flow." Perelman declined to accept the award or to appear at the congress, stating: "I'm not interested in money or fame, I don't want to be an advisor for an official or a son." On 22 December 2006, the scientific journal Science recognized Perelman's proof of the Poincaré conjecture as the scientific "breakthrough of the year", the first such recognition in the field of mathematics.

On 18 March 2010, it was announced that he had met the criteria to receive the first Clay Millennium Prize for resolution of the Poincaré conjecture. On 1 July 2010, he turned down the prize of one million dollars. He additionally turned down the prestigious prize of the European Mathematical Society.

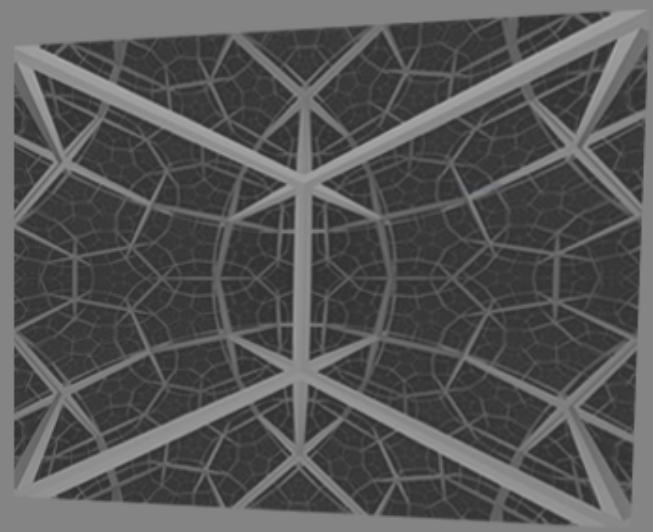


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Perelman's work on the Poincaré conjecture was a landmark achievement in topology. He proved that every simply connected, closed 3-manifold is homeomorphic to a 3-sphere. This result is considered one of the most important in the field of topology.



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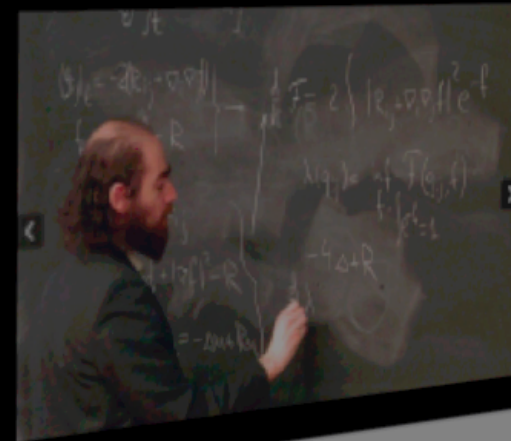
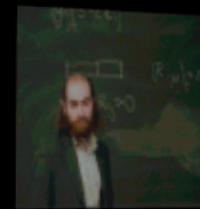
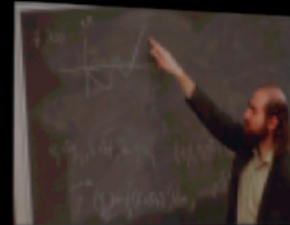
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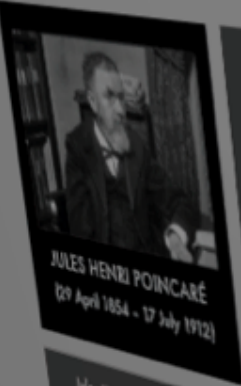
Grigori Yakovlevich Perelman is a Russian mathematician who made landmark contributions to Riemannian geometry and geometric topology before his presumed withdrawal from mathematics.

In 1994, Perelman proved the soul conjecture. In 2003, he proved Thurston's geometrization conjecture. This consequently solved in the affirmative the Poincaré conjecture, posed in 1904, which before its solution was viewed as one of the most important and difficult open problems in topology.

In August 2006, Perelman was awarded the Fields Medal [1] for "his contributions to geometry and his revolutionary insights into the analytical and geometric structure of the Ricci flow." Perelman declined to accept the award or to appear at the congress, stating: "I'm not interested in money or fame; I don't want to be on display like an animal in a zoo." On 22 December 2006, the scientific journal Science recognized Perelman's proof of the Poincaré conjecture as the scientific "Breakthrough of the Year", the first such recognition in the area of mathematics.

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JULES HENRI POINCARÉ
(29 April 1854 - 17 July 1912)

Jules Henri Poincaré foi um Matemático Francês, Físico Teórico, Engenheiro e Filósofo.

Ele é citado como o Último Universalista por Eric Temple Bell, pois teve atuação significativa em todos os campos da Matemática durante a sua carreira.

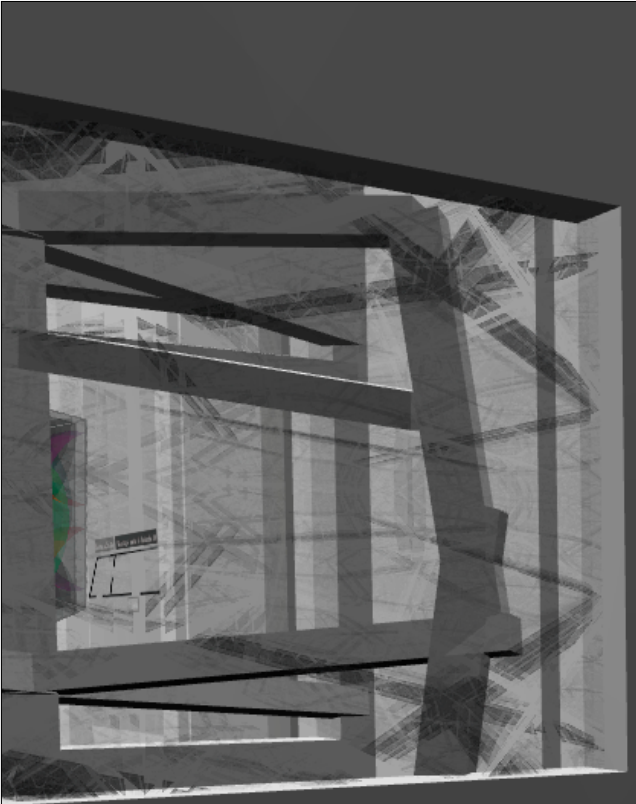
Como Matemático e Físico, ele é responsável por inúmeras contribuições fundamentais para a matemática pura e aplicada, física matemática e mecânica celeste.

He was responsible for formulating the Poincaré conjecture, which was one of the most famous unsolved problems in mathematics until it was solved in 2002-2003. In his research on the three-body problem, Poincaré became the first person to discover a chaotic deterministic system which laid the foundations of modern chaos theory. He is also considered to be one of the founders of the field of topology.

Poincaré made clear the importance of paying attention to the invariance of laws of physics under different transformations, and was the first to present the Lorentz transformations in their modern symmetrical form. Poincaré discovered the remaining relativistic velocity transformations and recorded them in a letter to Dutch physicist Hendrik Lorentz (1852-1928) in 1905. This he obtained perfect invariance of all of Maxwell's equations, an important step in the formulation of the theory of special relativity.



Jules Henri Poincaré reading a book by Gabriel, 1904-1905



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He was a pioneer in the
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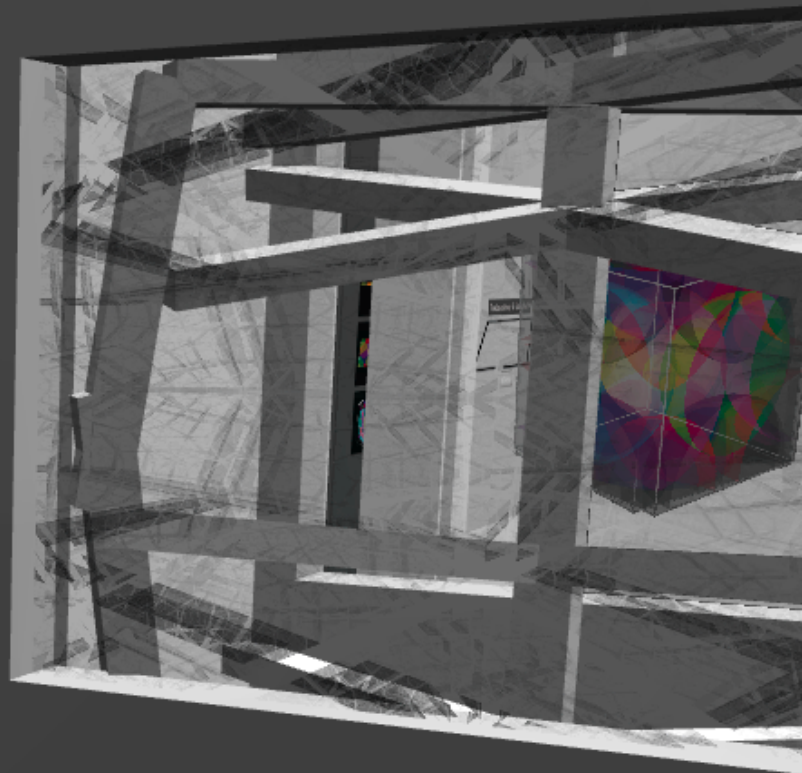
(24 de Março de 1892
- 22 de Junho de 1977)

Matemático Americano
mais conhecido pelo
seu trabalho sobre o
cálculo das variações.



Ele dedicou a maior parte da sua carreira a um único tópico, conhecido como Teoria de Morse, que atualmente configura um ramo importante da topologia diferencial. Além disso, a Teoria de Morse desempenha um papel fundamental na Física Matemática moderna.

Em 1983, Morse foi agraciado com o Prêmio Bôcher Memorial pelo seu trabalho na área de Análise Matemática.



variedades
dimensão 3



**GRIGORI YAKOVLEVICH
PERELMAN**

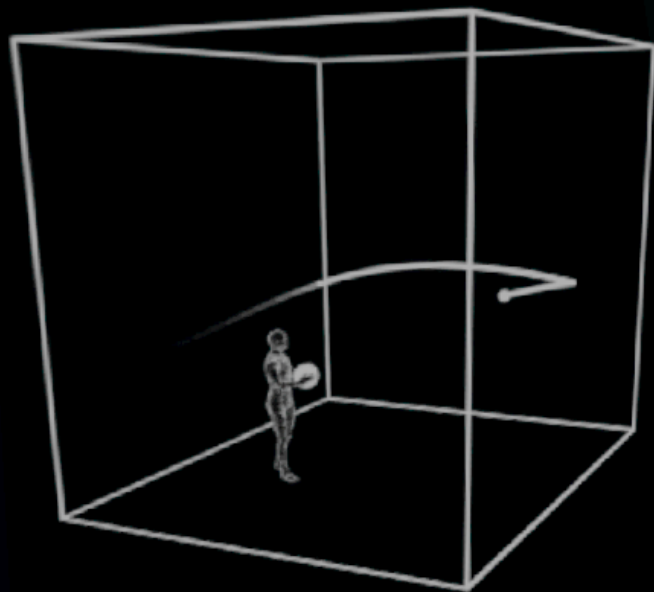
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Grigori Yakovlevich Perelman is a Russian mathematician who made landmark contributions to Riemannian geometry and geometric topology before his presumed withdrawal from mathematics.

Imagem por: [illegible]
Fonte: [illegible]

Visualização Interior de variedades
de Dimensão 3

Domínio Fundamental



Visão interior



io3



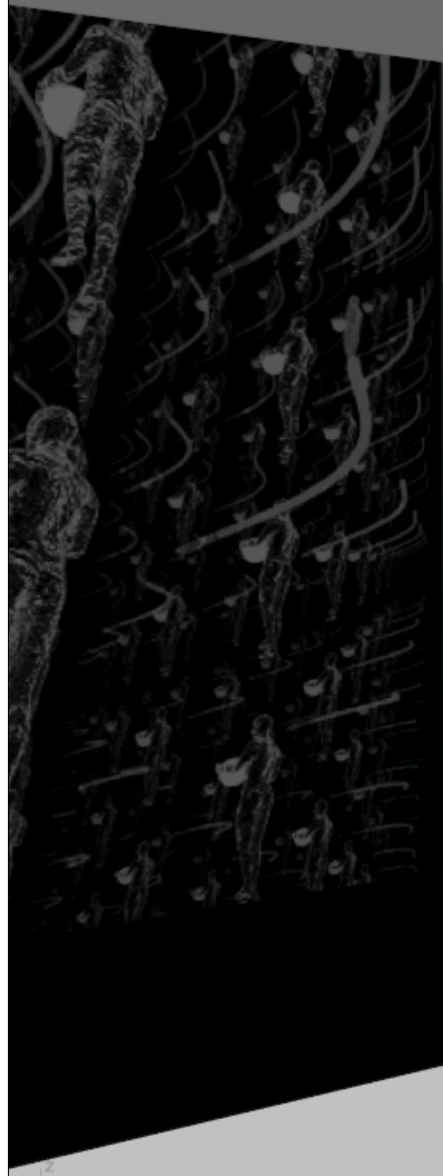
escolha sua geometria



e controle a partícula
na 3-variedade



terior



Handwritten mathematical notes and diagrams on a dark board, including various symbols and equations.

Teoremas e Citações

Classificação de Variedade de Dimensão 3

Visão interior das oito geometrias de Thurston

Uma esfera simplesmente conexa é topologicamente equivalente a uma 3-esfera.
Casos do Teorema que não podem ser reduzidos a um ponto.

S^3			$S^2 \times I$
E^3			HP^3
HP^3			Nil^3
$S^2 \times S^1$			Sol^3

Toda variedade ao longo de variedades de deformação em E^3 , HP^3 , Nil^3 , Sol^3



Porelman

Group Theory
Chapman - Group 5

Hermann Day by Hermann (1993)
"to see why a manifold deformation"

$R^2 \cong \mathbb{R}^2$
 $(\mathbb{R}^2 \rightarrow \mathbb{R}^2) \times (\mathbb{R}^2 \rightarrow \mathbb{R}^2) \times (\mathbb{R}^2 \rightarrow \mathbb{R}^2)$
 $+ 2(\mathbb{R}^2 \rightarrow \mathbb{R}^2) \times \mathbb{R}^2 + \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2$

Exterior products are linear when?
(take a base, the top two don't have orientation)



How to continue through the singularity?
Hamilton (18, 19), periodicity in discrete branching



Integrate: Geometric description of a path
in a manifold (or a surface)



How do we find the fundamental group?
There are two generators in the fundamental group



Now scale so $k=1$ on left or right
But take a limit of these along a relationship geometric relation.

$F(x,y) = \int_0^1 (x - y) dx$
 $(0,1) \rightarrow (1,0)$
 $(1,0) \rightarrow (0,1)$
 $(0,1) \rightarrow (0,0)$
 $(0,0) \rightarrow (0,1)$

Look at the numerical form of the F
Fundamental group

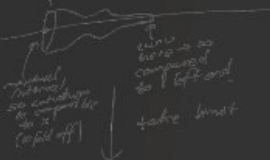
$V(x,y) = \int_0^1 (x^2 - y^2) dx$
 $(0,1) \rightarrow (1,0)$
 $(1,0) \rightarrow (0,1)$
 $(0,1) \rightarrow (0,0)$
 $(0,0) \rightarrow (0,1)$

In dimension 2, we are interested in π_1
which is not built in dimension 3.

Now some physical applications:
The fundamental is used in string theory

$F(x,y) = \int_0^1 (x + y) dx$
 $(0,1) \rightarrow (1,0)$
 $(1,0) \rightarrow (0,1)$
 $(0,1) \rightarrow (0,0)$
 $(0,0) \rightarrow (0,1)$

Higher symmetry with an arbitrary number of dimensions
but for fun it seems unrelated to \mathbb{R}^2



So now we know
like a cylinder

So take the cylinder as long
as possible.



spread out
like a comment
if \mathbb{Z} which are like
numbers + structure together

Now operators will be seen
obviously



Now scale so $k=1$ on left or right
But take a limit of these along a relationship geometric relation.

Proof
if k is small ≈ 0

$L(x,y) = \int_0^1 (x^2 - y^2) dx$
 $(0,1) \rightarrow (1,0)$
 $(1,0) \rightarrow (0,1)$
 $(0,1) \rightarrow (0,0)$
 $(0,0) \rightarrow (0,1)$

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$(0,1) \rightarrow (1,0)$
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 $(0,0) \rightarrow (0,1)$

Geometria Top



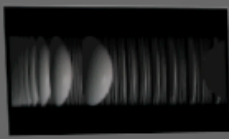
nas e Citações

ção de Variedade de Dimensão 3

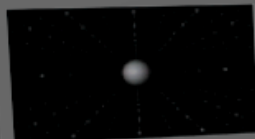
Visão interior das oito geometrias de Thurston



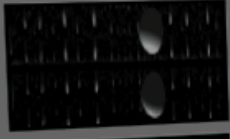
$S^3 \times E^1$



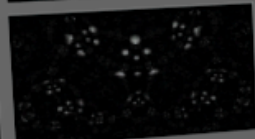
$H^3 \times E^1$



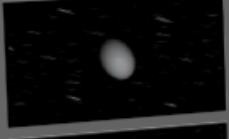
Nil



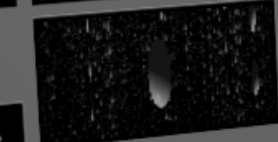
Sol



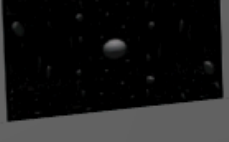
E^3



H^3



$H^3 \times E^1$



SL_2

Nil

Sol

$S^2 \times E^1$

S^2

Teorema de Geometrização
(Thurston, Palestras 2002)

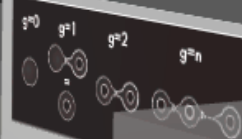
Toda variedade tridimensional pode ser cortada ao longo de superfícies e decomposta em variedades de dimensão 3 passíveis de deformação em uma das seguintes geometrias:

E^3 H^3 $H^3 \times E^1$ SL_2 Nil Sol $S^2 \times E^1$ S^2

Classificação de Superfícies

Teorema de Classificação
(A.F. Moebius 1839 e C. Jordan 1866)

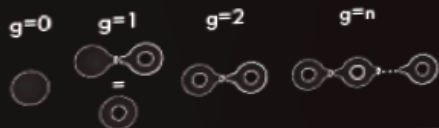
Toda superfície fechada no espaço pode ser deformada para corresponder a uma soma conexa de g toros, para $g \geq 0$.



Classificação de Superfícies

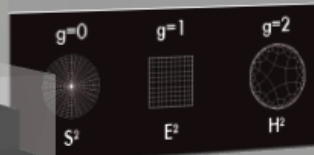
Teorema de Classificação
(A.F. Moebius 1896 e C. Jorda 1866)

Toda superfície fechada no espaço pode ser deformada para corresponder a uma soma conexa de g toros, para $g \geq 0$.



Teorema de Uniformização
(H. Poincaré, P. Koebe 1907)

Qualquer superfície fechada e orientável admite uma das estruturas geométricas: esférica (S^2), plana (E^2) ou hiperbólica (H^2).



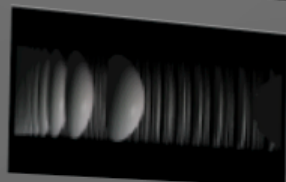
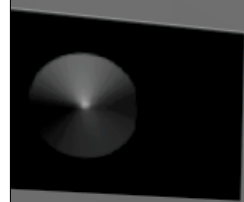
“ Uma geometria não é mais verdadeira que outra, ela somente pode ser mais conveniente... ”

H. Poincaré, La Science e L'Hypothèse

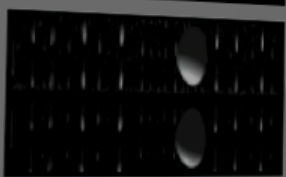
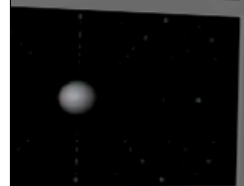
As e Citações

de Variedade de Dimensão 3

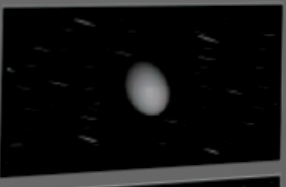
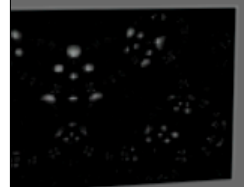
interior das oito geometrias de Thurston



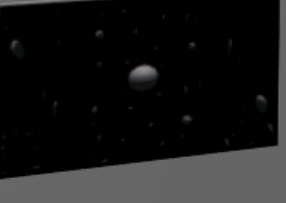
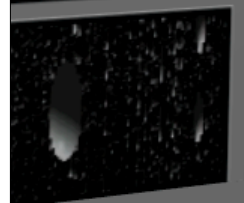
$S^2 \times E^1$



$H^2 \times E^1$



Nil



Sol

Teorema de Geometrizaç o
(Thurston, Poincar e 2002)

Toda variedade tridimensional pode ser cortada
ao longo de superf cies e decomposta em
variedades de dimens o 3 passíveis de
deformaç o em uma das seguintes geometrias:

E^3

H^3

$H^2 \times E^1$

SL_2

Nil

Sol

$S^2 \times E^1$

S^3

Teorema de Classificaç o
(A.F. Moebius 1896 e C. Jorda 1866)

Toda superf cie fechada no
espaço pode ser deformada
para corresponder a uma soma
conexa de g toros,
para $g \geq 0$.

$g=0$

$g=1$

$g=2$

$g=n$



Classificaç o



Teoremas e Conjecturas

Classificação de Variedade de Dimensão 3



Conjectura
(H. Poincaré, 1900)

Toda 3-variedade fechada e simplesmente conexa é homeomorfa à 3-esfera.

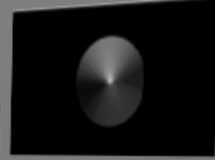
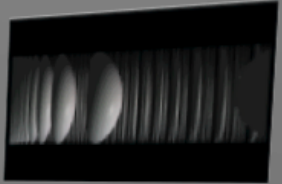
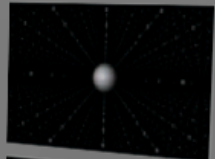
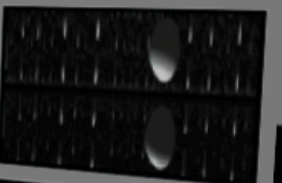
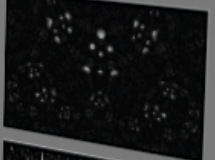
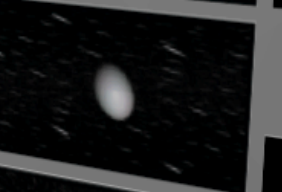
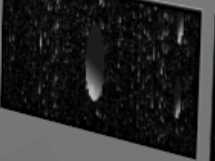
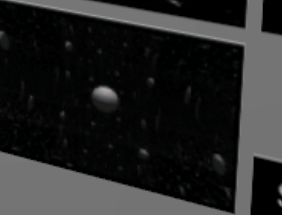
Toda cancha fechada no 2-esfera
pode ser contraída a um ponto.



Canchais no 2-toro que não podem ser
contraídos a um ponto.



Visão interior das oito geometrias de Thurston

S^3			$S^2 \times E^1$
E^3			$H^2 \times E^1$
H^3			Nil
SL_2			Sol

Saiba mais

LIVROS



The Physics Cookbook: In Search of the Shape of the Universe
David O'Neil, 2007



The Shape of Space: An Exploration of Mathematical Universes
Jull Winkler
Math Resources - National Museum of Mathematics, Fevereiro 2012



Euclid: A Renaissance of Many Dimensions
Edwin A. Abbott, 1992

PALESTRAS



The Geometry of 2-Manifolds
John J. Rotman
Harvard Science Center Research Lecture Series, Outubro 2006



The Shape of Space: An Exploration of Mathematical Universes
Jull Winkler
Math Resources - National Museum of Mathematics, Fevereiro 2012



Shape of Space
Jull Winkler
Columbia University World and I, Fevereiro 2012

VÍDEOS



Dimensiones
Anja Jans, Bianca Glyn, Annelien Akkers, 2010



The Shape of Space (Geometry Center) - Video
Science Museum e Dutch Museum, 1997



Geodesics and Waves
Koenrad Polster, Michael Schreiber, Martin Rufers, Christian Teuber, 1997

ORGANIZAÇÕES



The Geometry Center
http://www.geometrycenter.edu/



Clay Mathematics Institute
http://www.claymath.org/



Visite o site da exposicao na Internet para uma obter uma lista completa de referencias sobre o assunto:
<http://visgraf.impa.br/olhar3d>

...sistema de coordenadas ou mais de uma variável, ou cuja estrutura seja inicialmente definida em termos globais, provavelmente vai requerer para a sua solução considerações de topologia e teoria dos grupos.

Na resolução de tais problemas, a análise clássica aparece frequentemente como um instrumento local, integrado ao problema como um todo através da topologia ou teoria dos grupos.

(M. Morse, *Calculus of variations in the large*, 1924)

Saiba mais

VÍDEOS



Dimensiones
Joe Gray, Evanow City, Avellan Adams, 2010



The Shape of Space (Geometry Center) - Video
Sussex Museum of Art & Design, 1997



Overlays and Wires
Klaus Fuchs, Martin Schreiber, Jürgen Schöfer, Christian Tölgel, 1997

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The Geometry Center
<http://www.geometrycenter.org/>



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<http://www.clayinstitute.org/>

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na lista completa de referências sobre o assunto:
isgraf.impa.br/olhar3d

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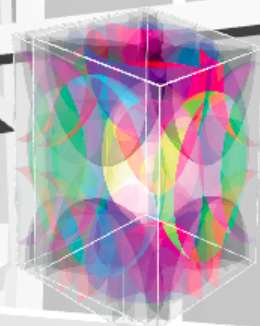
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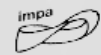
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